Foundations of XML Types: Suggested Answers

Trees and Tree Automata

Q1. Give a bottom-up deterministic tree automaton that recognizes the tree language L composed of the two trees below:

A1. A sample bottom-up deterministic tree automaton B that recognizes L:

$$\begin{array}{rl}
\text{Alphabet}(B) & : \{a^{(2)}, b^{(0)}, c^{(0)}\} \\
\text{States}(B) & : \{q_a, q_b, q_c\} \\
\text{Final}(B) & : \{q_a\} \\
\text{Rules}(B) & : \{(q_b, q_c) \xrightarrow{a} q_a, \quad (q_c, q_b) \xrightarrow{a} q_a, \quad \epsilon \xrightarrow{b} q_b, \quad \epsilon \xrightarrow{c} q_c\}
\end{array}$$

Q2. Bottom-up tree automata seen during the course traverse trees from the leaves to the root. In a similar manner, one may define top-down tree automata that recognize trees by going in the opposite direction: from the root to the leaves. Specifically, a top-down tree automaton A consists in:

$$\begin{array}{rl}
\text{Alphabet}(A): & \text{finite alphabet of symbols} \\
\text{States}(A): & \text{finite set of states} \\
\text{Rules}(A): & \text{finite set of transition rules} \\
\text{Initial}(A): & \text{finite set of initial states } (\subseteq \text{States}(A)) \\
q_{\text{acc}} \in \text{States}(A): & \text{final state}
\end{array}$$

There are two major differences with automata seen during the course:

- transition rules are either of the form: $q \xrightarrow{a} (q_1, q_2)$ where $q, q_1, q_2 \in \text{States}(A)$ and $a \in \text{Alphabet}(A)$ or of the form $q \xrightarrow{a} q_1$ for leaves.
- a tree is accepted if and only if there exists a run for which all the leaves are labeled with q_{acc}.

Give a top-down tree automaton that recognizes L.

A2. A sample top-down tree automaton T that recognizes L:

$$\begin{array}{rl}
\text{Alphabet}(T): & \{a^{(2)}, b^{(0)}, c^{(0)}\} \\
\text{States}(T): & \{q_a, q_{\text{acc}}\} \\
\text{Initial}(T): & \{q_a\} \\
q_{\text{acc}} \in \text{States}(T): & \text{final state} \\
\text{Rules}(T): & \{q_a \xrightarrow{a} (q_b, q_c), \quad q_a \xrightarrow{a} (q_c, q_b), \quad q_b \xrightarrow{b} q_{\text{acc}}, \quad q_c \xrightarrow{c} q_{\text{acc}}\}
\end{array}$$

Q3. Do you see any interest of top-down tree automata in the context of XML stream processing where XML documents are sequentially parsed (only once) and processed on the fly? Explain.

A3. A top-down tree automaton can be used to implement on-the-fly validation of an XML stream against a given schema. In this context, nodes of an XML document are scanned, parsed, and processed on-the-fly starting from the root and in the order of a depth-first tree traversal. Top-down automata are more appropriate in this setting than bottom-up automata. This is because bottom-up automata have to wait for the full document (leaves) in order to be able to start validation. On the opposite, some transitions of top-down automata can be triggered without having to wait for the full document, so that they can be used to detect errors earlier with a stream (i.e., an incomplete document).

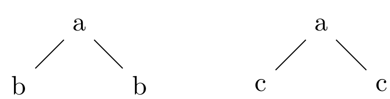
Q4. A top-down tree automaton is deterministic iff (1) there is at most one initial state and (2) for each $q \in \text{States}(A)$ et $a \in \text{Alphabet}(A)$ there is at most one rule $q \xrightarrow{a} (q_1, q_2)$ (intuitively, there is at most one possible transition for each state and symbol).
Is it possible to give a deterministic top-down tree automaton that recognize L? Either give it or justify.

A4. It is not possible. Indeed, let's try to build a deterministic top-down tree automaton DT that recognizes L:

$$\begin{aligned} \text{Alphabet}(DT) &: \{a^{(2)}, b^{(0)}, c^{(0)}\} \\ \text{States}(DT) &: \{q_a,\ q_{\text{acc}}\ \} \\ \text{Initial}(DT)&: \ \{\ q_a\} \\ q_{\text{acc}} \in \text{States}(DT) &: \ \text{final state} \end{aligned}$$

The impossible part is to define $\text{Rules}(DT)$:

DT must be deterministic so we have no other choice then putting only one transition rule for q_a and a, such as: $q_a \xrightarrow{a} (\mathrm{q}, \mathrm{q})$. Then, while still keeping DT deterministic, the only thing we can do is to add the rules $\mathrm{q} \xrightarrow{b} q_{\text{acc}}$ and $q \xrightarrow{c} q_{\text{acc}}$. However, if we define $\text{Rules}(DT) = \{q_a \xrightarrow{a} (\mathrm{q}, \mathrm{q}) \quad q \xrightarrow{b} q_{\text{acc}}, \quad q \xrightarrow{c} q_{\text{acc}}\}$ then DT also recognizes the two trees below:



These two trees are not part of L. There does not exist any deterministic top-down tree automaton that can recognize L (and only L).

Q5. It is known that non-deterministic bottom-up and non-deterministic top-down automata are equally expressive. From your answers to the previous questions, what can you conclude about the respective expressive power of deterministic bottom-up and deterministic top-down tree automata? Justify.

A5. If we sum up: the tree language L can be recognized by a bottom-up tree automata (see B above), and by a non-deterministic top-down tree automaton (see T above). However, no deterministic top-down tree automaton can recognize L (see previous question). Thus, deterministic top-down tree automata are strictly less expressive than non-deterministic top-down tree automata.