Efficient Static Analysis of XML Paths and Types

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Joint work with Nabil Layaïda and Alan Schmitt – INRIA, France

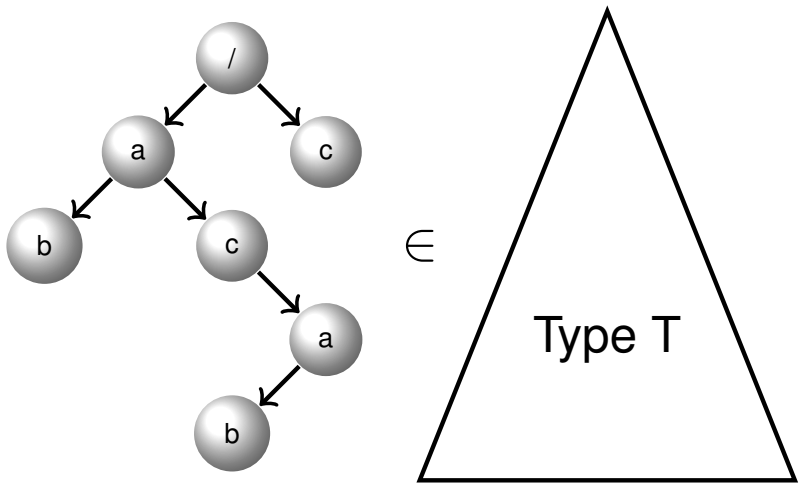
PLDI'07, San Diego, June 2007

Introduction

- More and more XML data
- **Objective:** ensuring safety and efficiency of programs that manipulate XML
- Two ways for processing XML:
  1. General purpose languages extended with librairies
  2. DSLs: e.g. **XSLT, XQuery** (W3C standards) that rely on **XPath**
- In both cases: static analysis of programs very hard (very complex to detect errors at compile-time)
- **This paper:** we solve important XML static analysis tasks by reduction to satisfiability of a new tree logic

Safety and Efficiency of Programs

- Programs that manipulate XML trees
- Analysis:
  - tree types (XML Schemas, DTDs)
  - queries (XPath)



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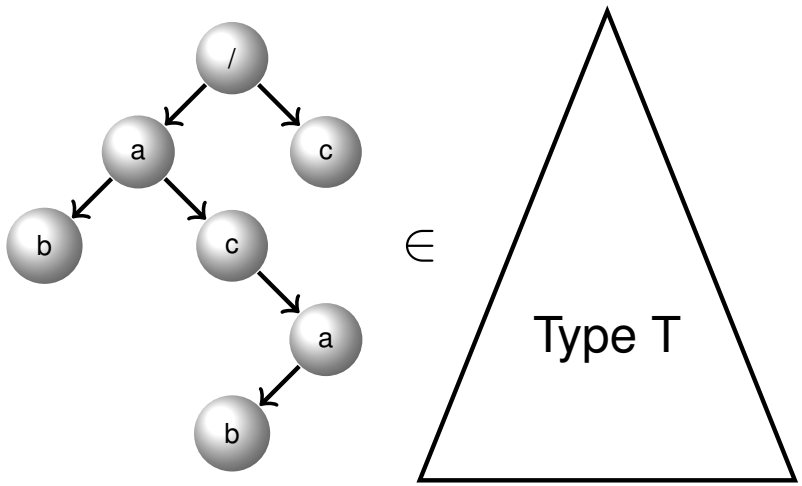

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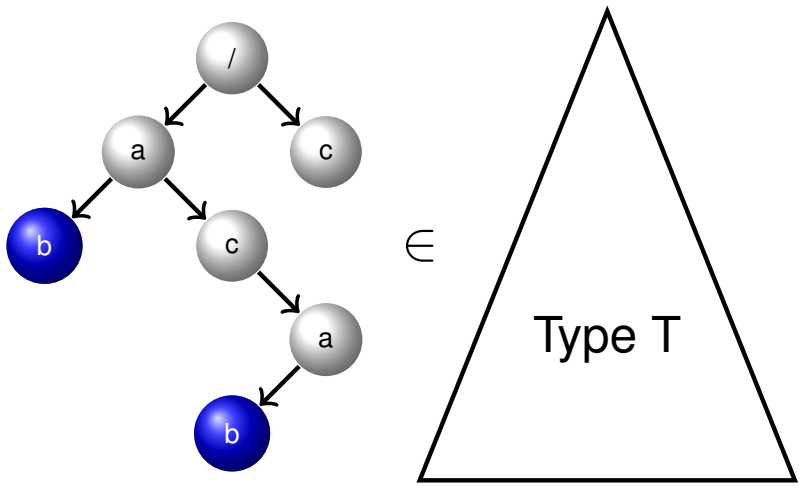

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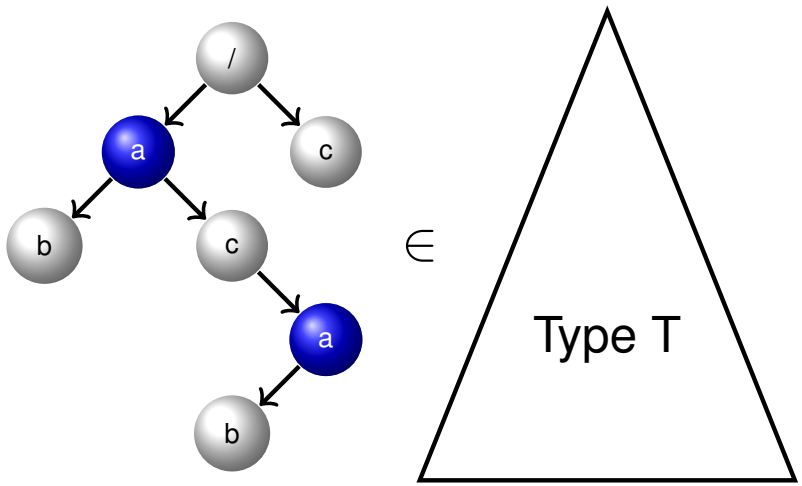

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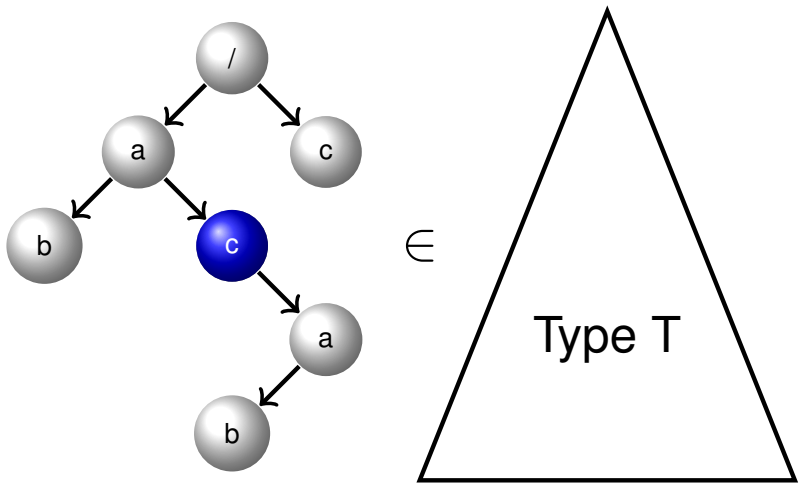

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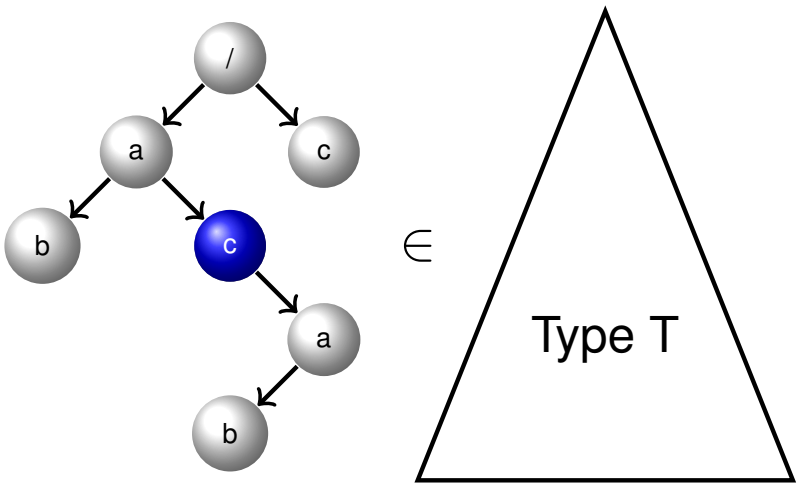

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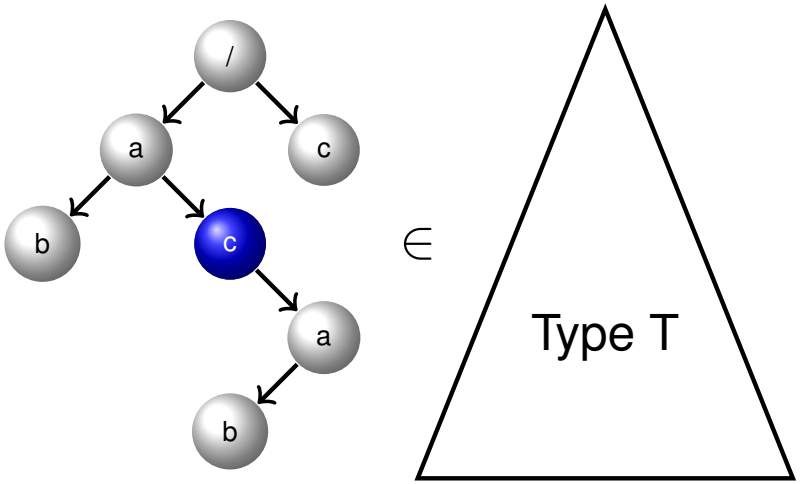

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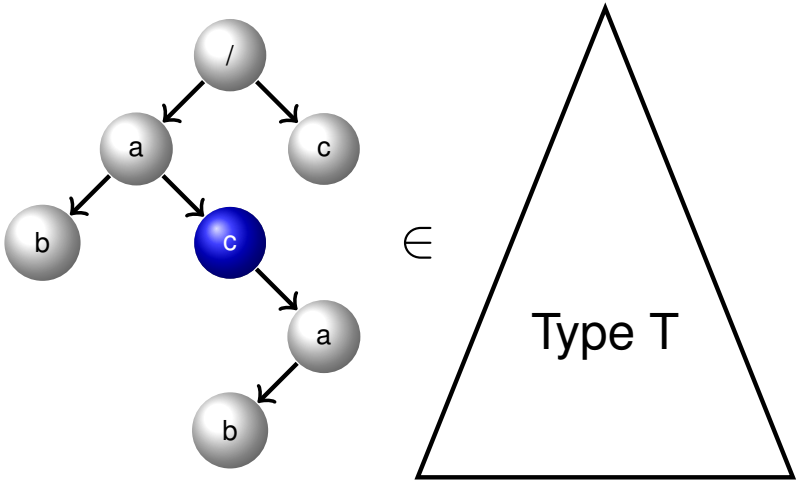

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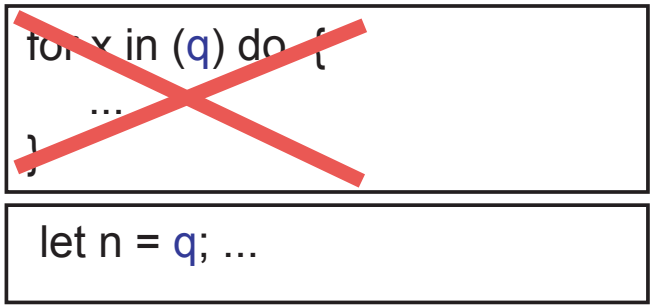

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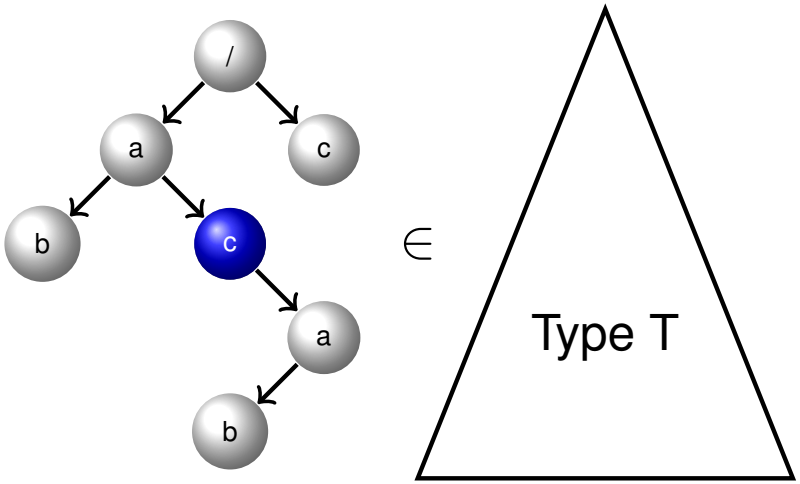

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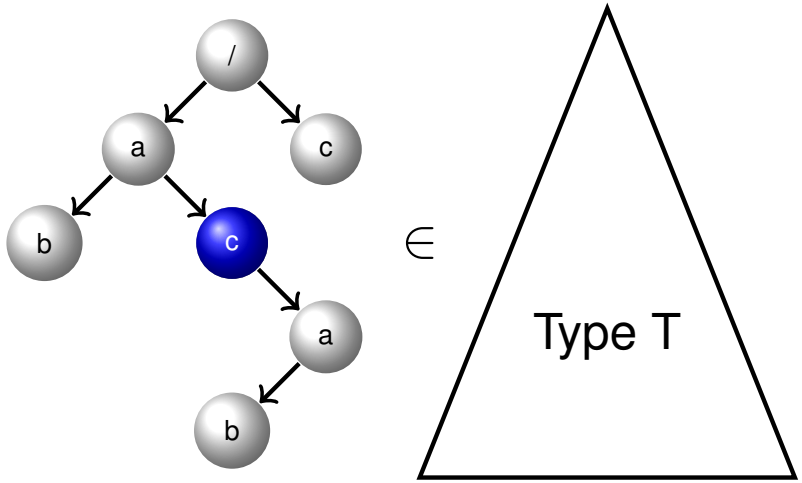

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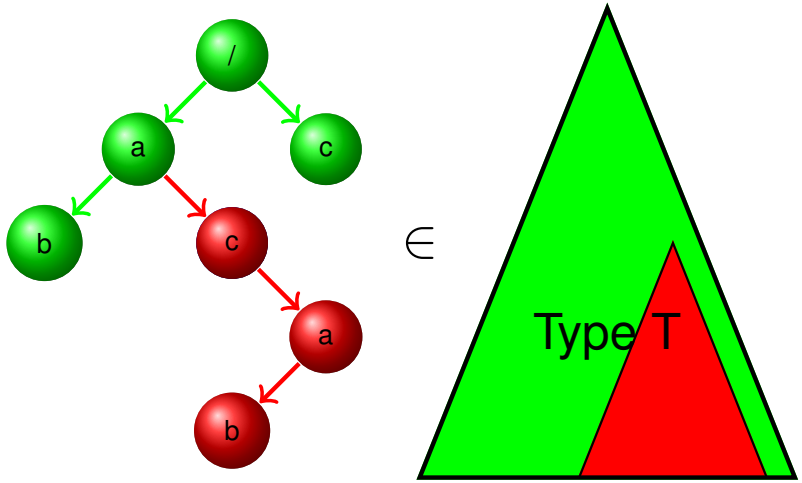

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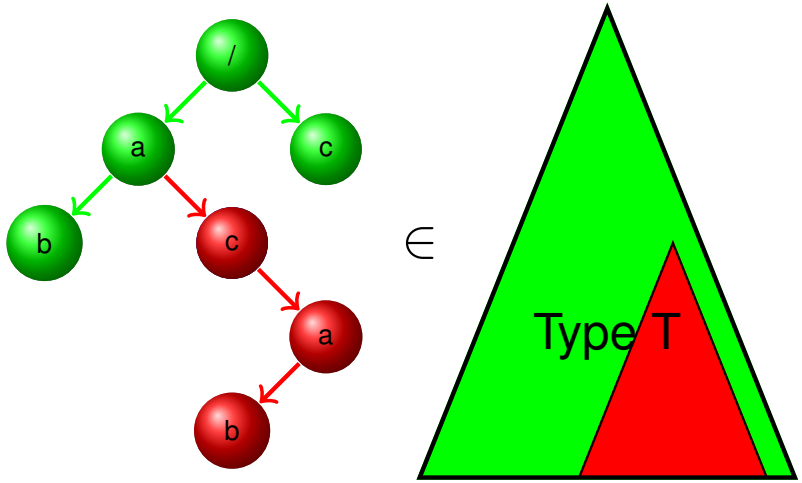

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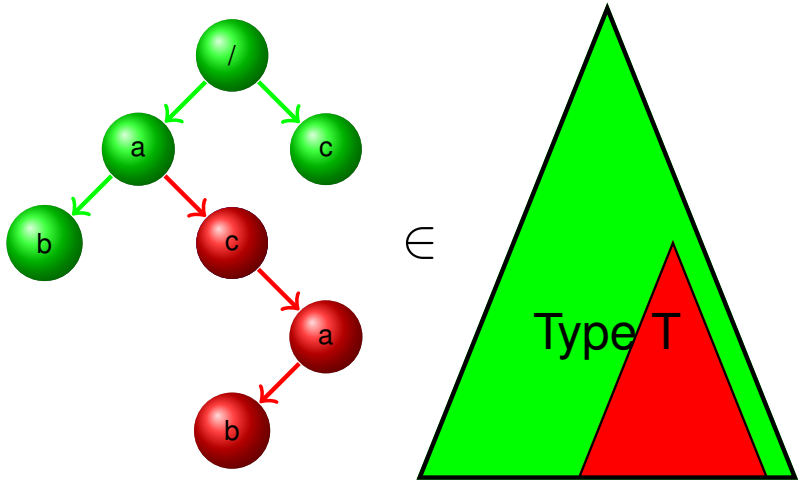

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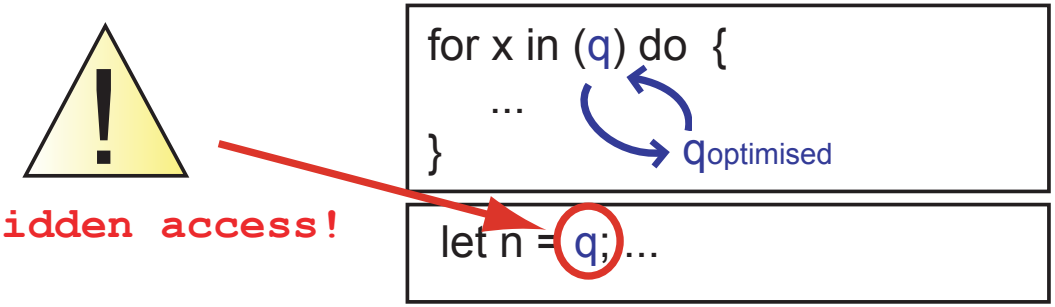

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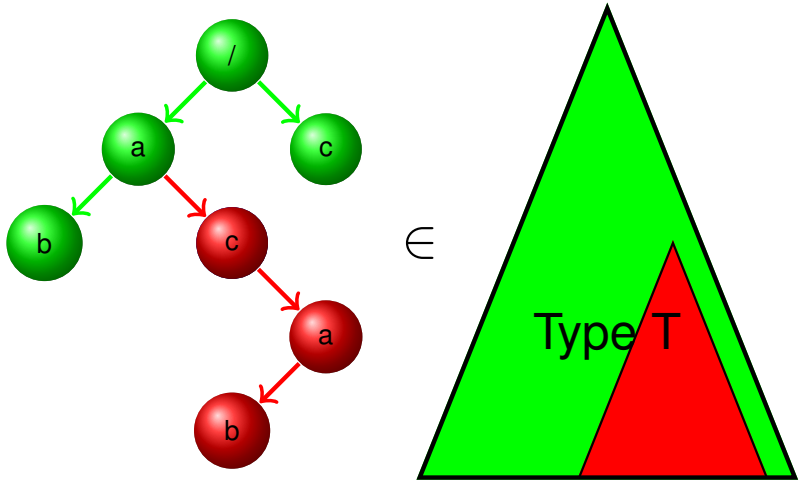

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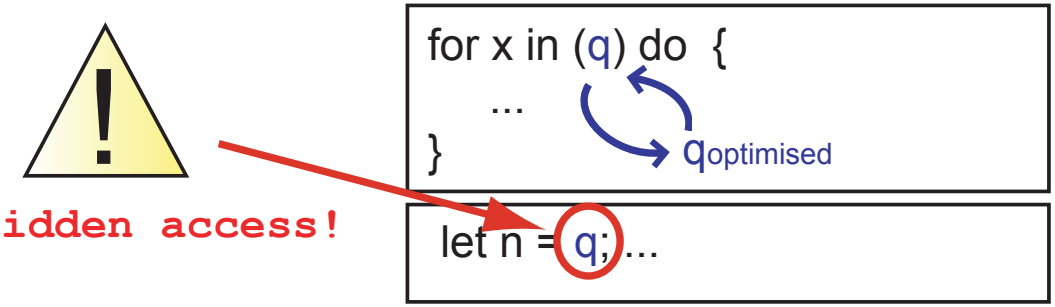

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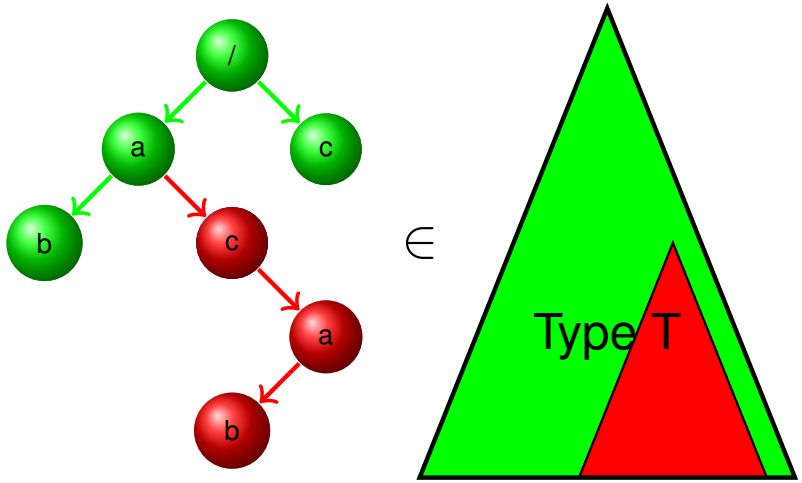

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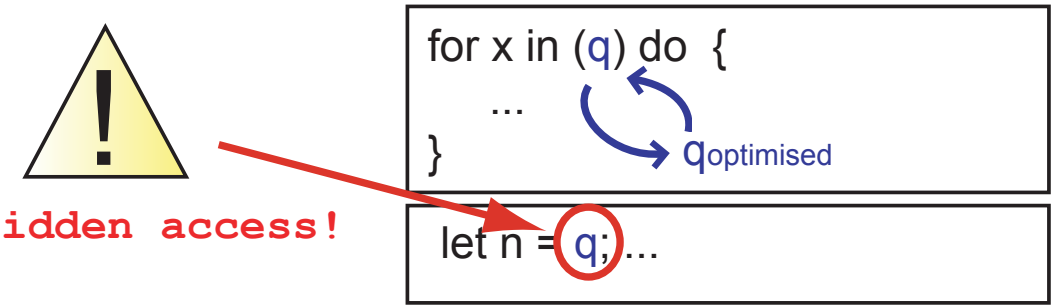

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XPath Static Analysis Tasks

Basic Tasks

1. XPath typing
2. XPath query comparisons
   - query containment, emptiness, overlap, equivalence

Main Applications

- Static analysis of host languages: error detection, optimization (static type-checkers, optimizing compilers)
- Checking integrity constraints in XML databases

Challenges

- Query comparisons and typing are undecidable for the complete XPath language

Open Questions

- What are the largest XPath fragments with decidable static analysis?
- Which fragments can be effectively decided in a compiler?
- Is there a generic algorithm able to solve all related XPath decision problems?

Difficulties

- Considered XPath operators and their combination (e.g., multidirectional navigation, recursion)
- Checking properties on a possibly infinite set of XML documents
- Very high computational complexity

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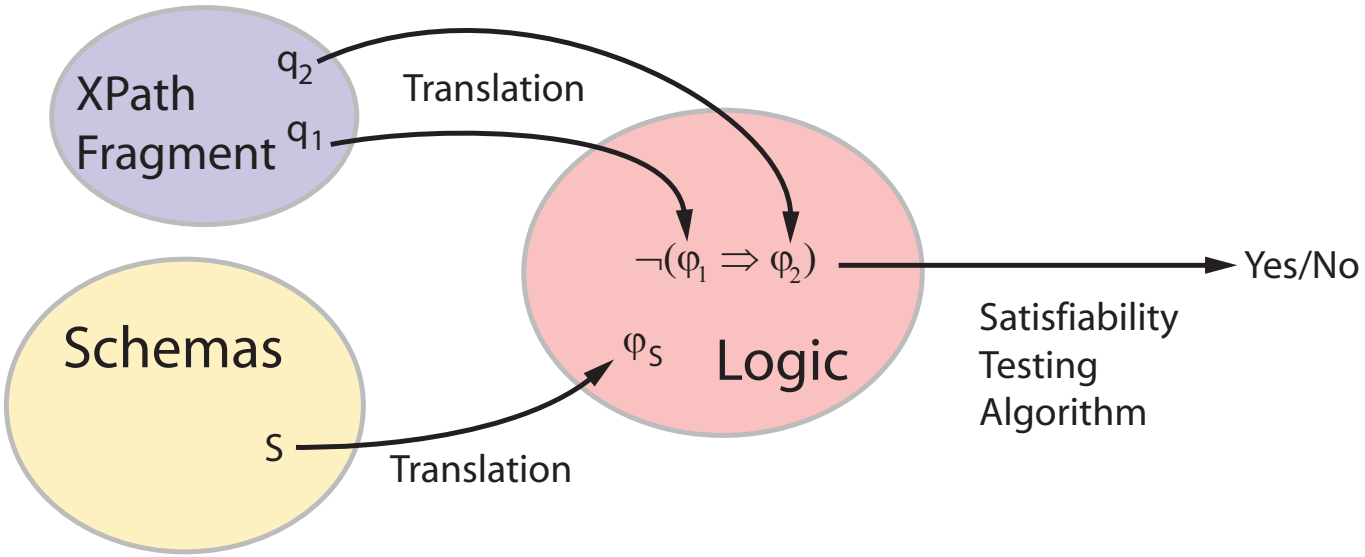
- Find an appropriate logic for reasoning on XML trees
- Formulate the problem into the logic and test satisfiability



counter-example

Critical Aspects

1. The logic must be expressive enough
2. The algorithm must be effective in practice for XML translations

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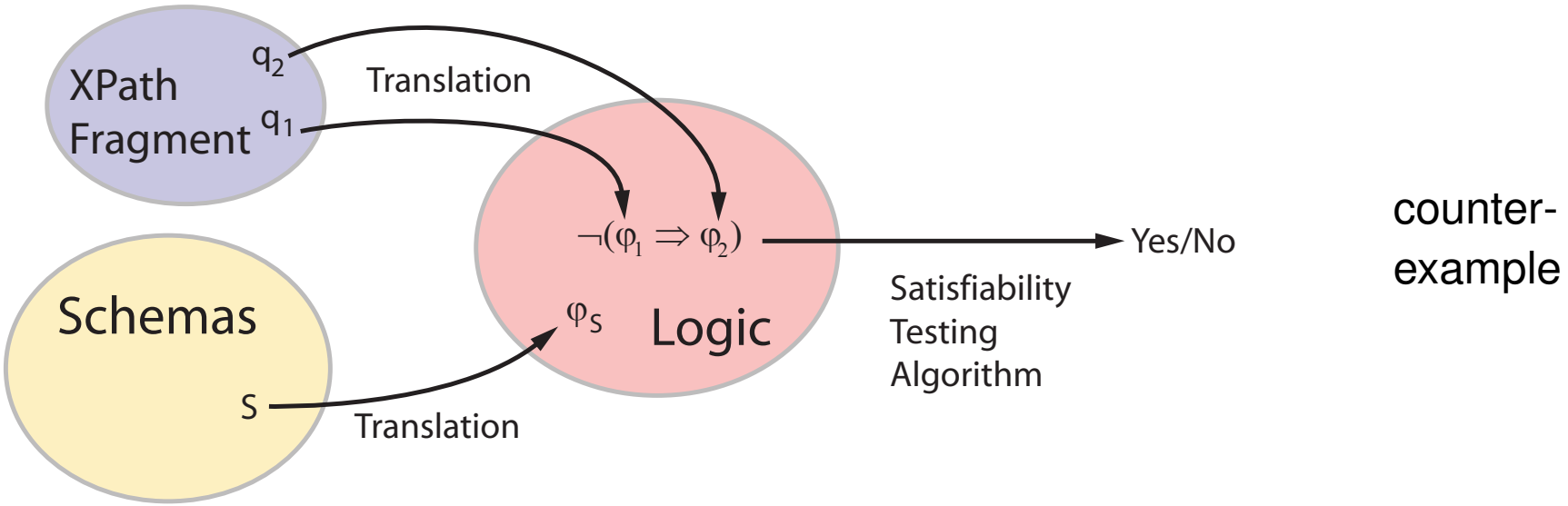

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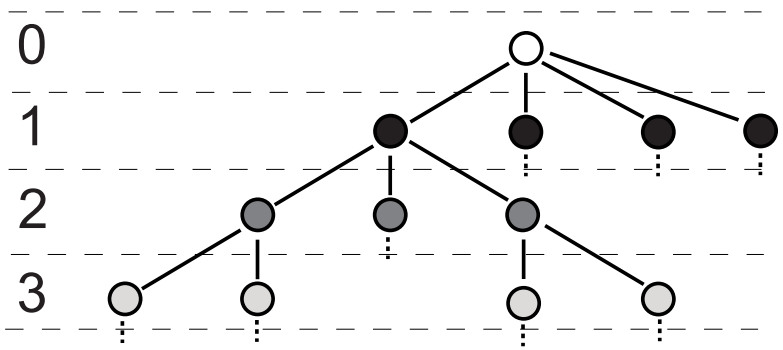
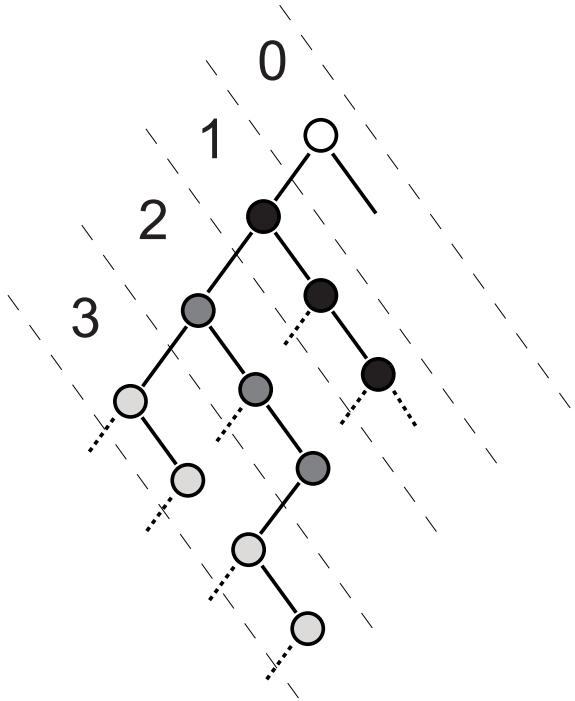
Models for XML Documents

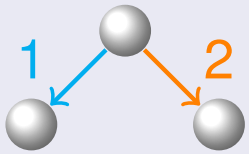
- Finite ordered binary trees, one label per node
- Bijective encoding of unranked trees as binary trees:





Formulas of the $\mathcal{L}_\mu$ Logic

- Programs $\alpha \in \{1, 2, \overline{1}, \overline{2}\}$ for navigating binary trees ($\overline{\overline{\alpha}} = \alpha$)



$\mathcal{L}_\mu \ni \varphi, \psi$	$::=$	formula
	$\top$	true
$\mid$	$\sigma \quad \mid \quad \neg\sigma$	atomic prop (negated)
$\mid$	$\textcircled{S} \quad \mid \quad \neg\textcircled{S}$	starting context (negated)
$\mid$	$\varphi \vee \psi$	disjunction
$\mid$	$\varphi \wedge \psi$	conjunction
$\mid$	$\langle\alpha\rangle\,\varphi \quad \mid \quad \neg\langle\alpha\rangle\,\top$	existential (negated)
$\mid$	X	variable
$\mid$	$\mu X.\varphi$	unary fixpoint
$\mid$	$\mu\overline{X_i.\varphi_i}$ in ψ	n-ary fixpoint

- Closed formulas

Semantics of $\mathcal{L}_\mu$

- The set of models of a formula φ is the set of finite binary trees for which φ is satisfied on some node

- $\mu Z.\varphi$: finite recursion
- $\{\overline{1}, \overline{2}\}$ required for forward axes!
- $\{1, 2\}$ required for reverse axes!
- Converse programs are crucial
- Almost full XPath can be translated (only variable counting constraints and data value comparisons left)
- Schemas can also be captured!

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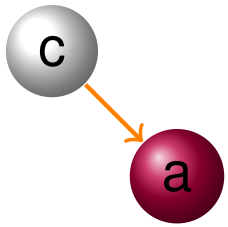

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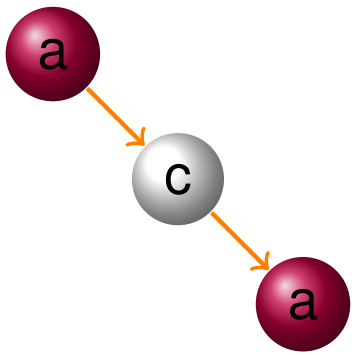

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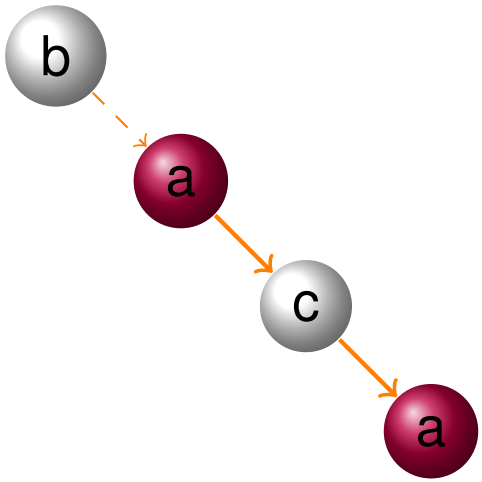

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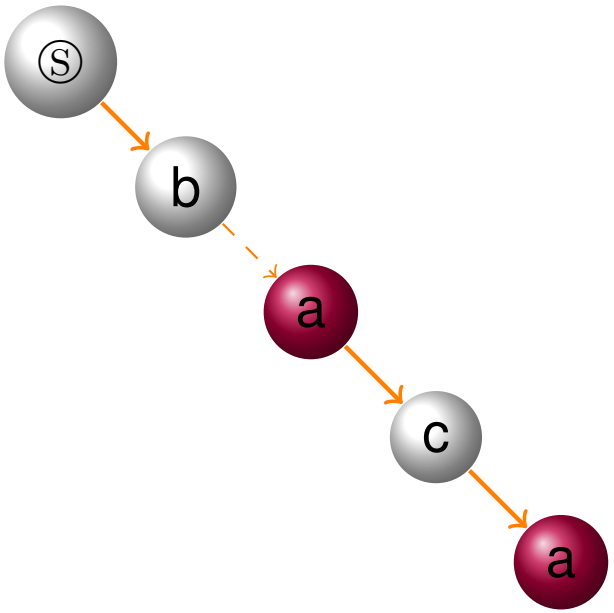

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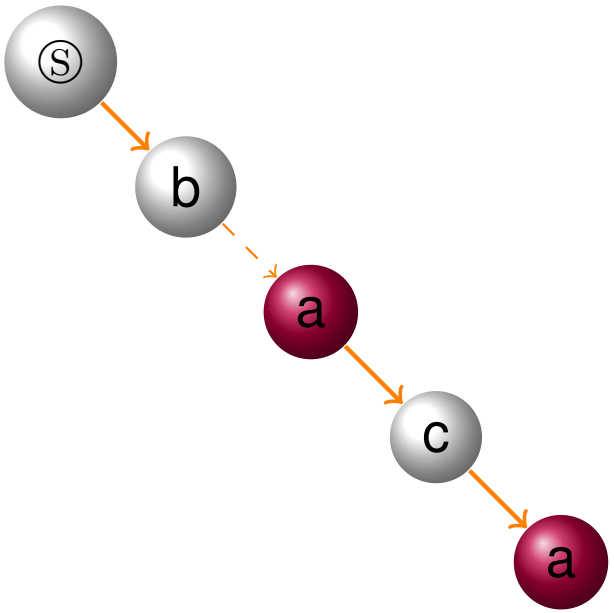

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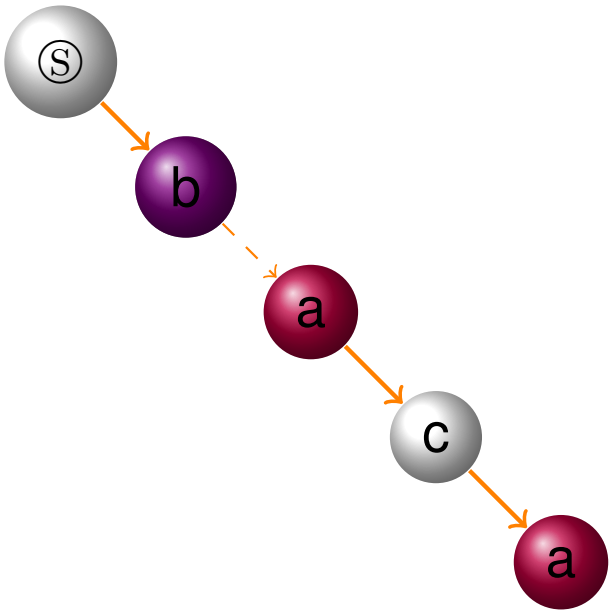
in $\mathcal{L}_\mu$: $b \wedge [\mu Y.\, \langle 2 \rangle\, (a \wedge (\mu Z.\, \langle \overline{2} \rangle\, \textcircled{s} \vee \langle \overline{2} \rangle\, Z))\, \vee \langle 2 \rangle\, Y]$



- $\mu Z.\varphi$: finite recursion
- $\{\overline{1}, \overline{2}\}$ required for forward axes!
- $\{1, 2\}$ required for reverse axes!
- Converse programs are crucial
- Almost full XPath can be translated (only variable counting constraints and data value comparisons left)
- Schemas can also be captured!

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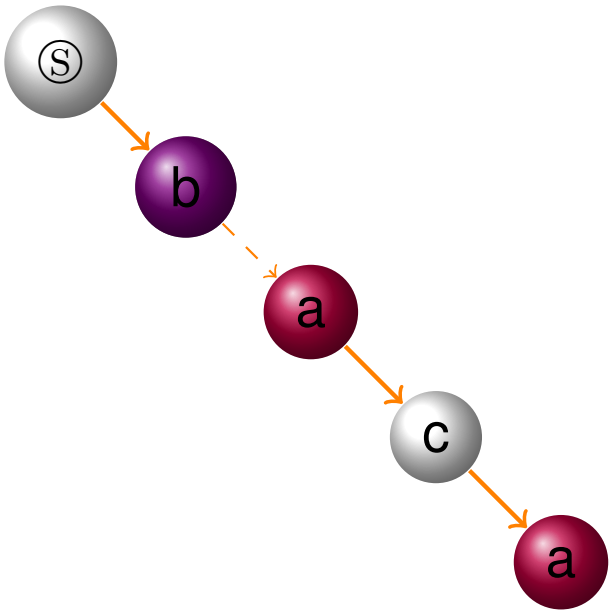

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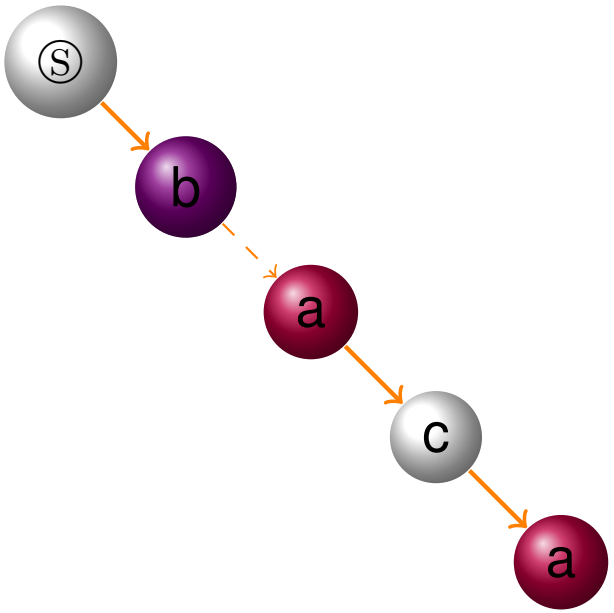

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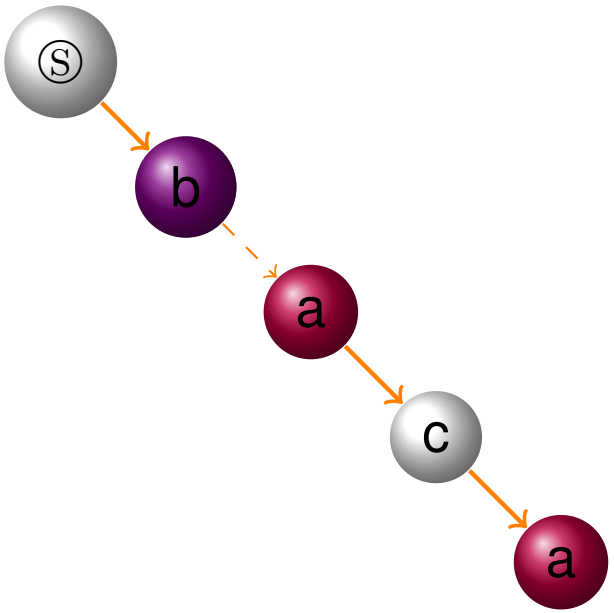

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Satisfiability-Testing Algorithm: Principles

Search for a Tree that Satisfies ψ

- ψ truth status can be determined from a few of its subformulas
- A node is a ψ-type (conjunction of formulas)

Bottom-up Construction of a Tree of ψ-types

- A set T of ψ-types is repeatedly updated (least fixpoint computation)
  - Initially: $\emptyset$
  - Step 1 : all possible leaves are added
  - Step i : all possible parent nodes of current nodes are added

Termination

- If ψ is present in some node, then ψ is satisfiable
- Otherwise, the algorithm terminates when no more node can be added

Correctness & Complexity

Theorem

The satisfiability problem for a formula $\psi \in \mathcal{L}_\mu$ is decidable in time $2^{O(n)}$ where n is the size of ψ.

Experimental Results

The First Implementation

- Able to handle such a large XPath fragment
- Able to handle schemas (regular tree types)

What Can Now Be Done

Time (s)	Solved Problems
< 0.5	Comparisons of XPath queries (XPathmark) without tree types
< 1	Medium tree types involved ($\approx$ 30 symbols, $\approx$ 20 variables) Example: W3C SMIL
< 3	Large tree types involved ($\approx$ 100 symbols, $\approx$ 400 variables) Example: W3C XHTML

Summary and Perspectives

A New Tree Logic

- Best balance known between expressiveness/complexity
- Translation of main XML concepts: linear
- Implementation already fairly efficient for static analysis

Future Work

- Extensions of the logic
  - Decidable data-value comparisons
  - Decidable counting constraints
- Type inference for XSLT/XQuery without output type annotations
- More applications in program analysis?
  - $\mathcal{L}_\mu$ is as expressive as MSO, and the solver is orders of magnitude faster than MONA...

Thank you!

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