Logics for XML

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Ph.D Defense – December 4th 2006

Outline

XML and Schemas

Extensible Markup Language (XML)

- A markup langage for representing tree structures
- Representation is independent from processing

Schemas

- Each application defines constraints on documents using a schema
- Several formalisms exist for defining schemas (e.g., DTD, XML Schema, Relax NG)

XPath: The Standard Query Language

- For navigating and extracting information from XML trees
- Evaluating an XPath query from a given context node returns a set of matching nodes

XPath Query Example

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child::division/child::group
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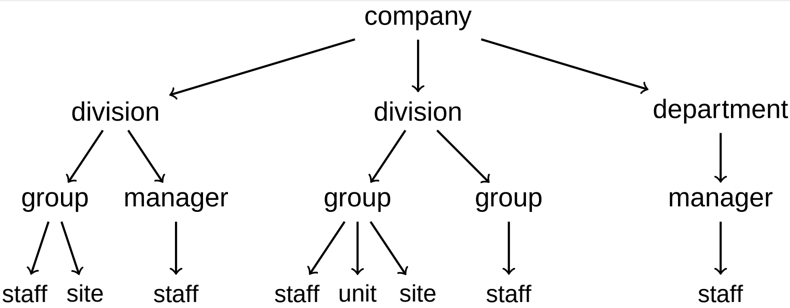

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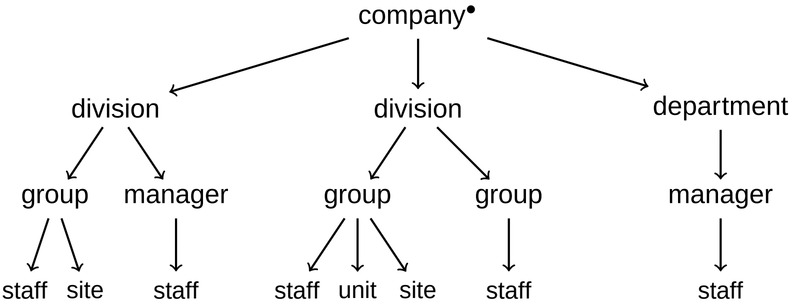

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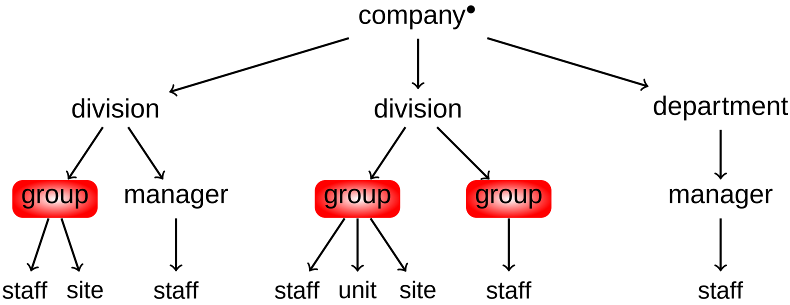

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XPath Query Example #2

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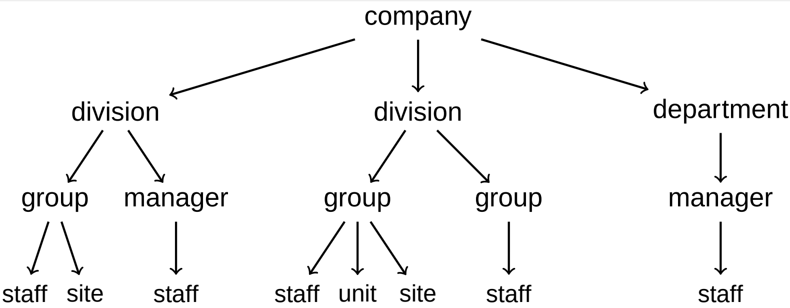

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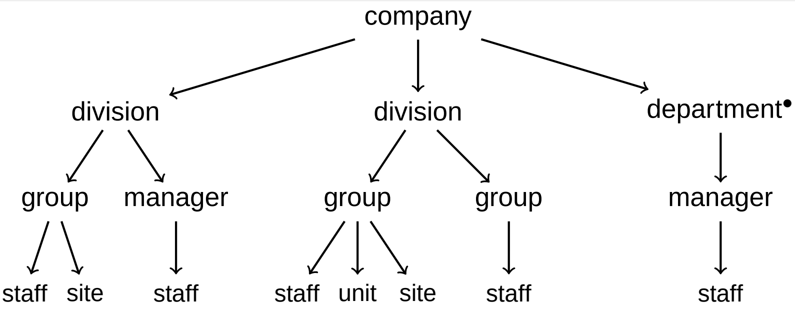

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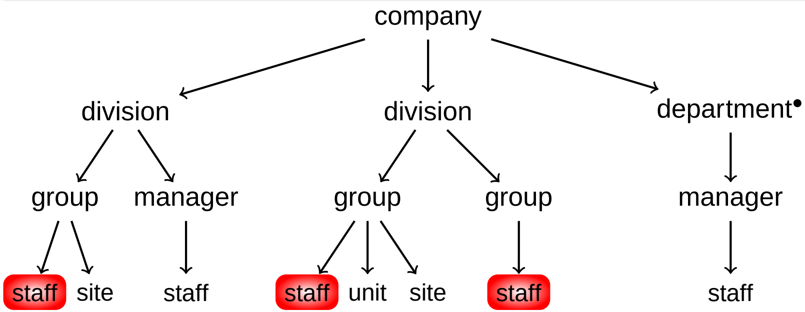

Motivation: Safe and Efficient XML Processing

- XPath plays a central role in key standards (e.g. XSLT, XQuery...)
- Static analysis of XPath has become crucial

Basic Static Analysis Tasks

1. XPath typing
2. XPath query comparisons
   - query containment, emptiness, overlap, equivalence

Main Applications

- Static analysis of host languages (e.g., type-checking of XSLT, XQuery), error-detection, optimization
- Checking integrity constraints in XML databases, XML security

- Objective: effectively analyzing XPath queries with schemas

Challenges

- Query comparisons and typing are undecidable for the complete XPath language

Open Questions

- What are the largest XPath fragments with decidable static analysis?
- Which fragments can be effectively decided in practice?
- Is there a generic algorithm able to solve all related XPath decision problems?

Difficulties

- Considered XPath operators and their combination (e.g., reverse axes, recursion)
- Checking properties on a possibly infinite set of XML documents
- Very high computational complexity

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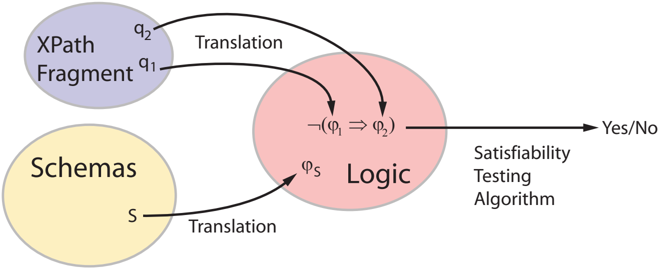
The Logical Approach: Overview

- Find an appropriate logic for reasoning on XML trees
- Formulate the problem into the logic and test satisfiability



Critical Aspects

1. The logic must be expressive enough
2. The algorithm must be effective in practice for XML translations

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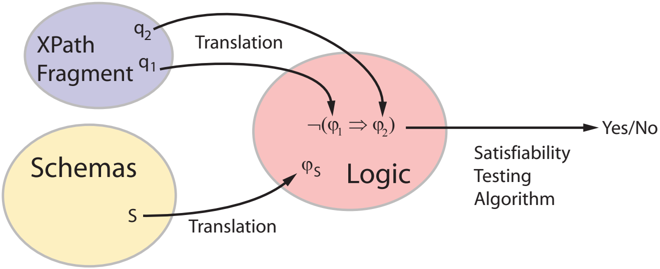

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A Large XPath Fragment

$$
\begin{array}{llcl}
\mathcal{L}_{\mathrm{XPath}} \ni e & ::= & & /p \\
 & & \mid & p \\
 & & \mid & e_1 \mid e_2 \\
 & & \mid & e_1 \cap e_2 \\
\mathit{Path}\ \ p & ::= & & p_1 / p_2 \\
 & & \mid & p[q] \\
 & & \mid & \mathrm{a}::\sigma \\
 & & \mid & \mathrm{a}::* \\
\mathit{Qualif}\ \ q & ::= & & q_1 \text{ and } q_2 \\
 & & \mid & q_1 \text{ or } q_2 \\
 & & \mid & \text{not } q \\
 & & \mid & p \\
 & & \mid & \cancel{\text{count}(p) = n} \\
 & & \mid & \cancel{p_1 = p_2} \\
\mathit{Axis}\ \ a & ::= & & \text{child} \\
 & & \mid & \text{self} \\
 & & \mid & \text{parent} \\
 & & \mid & \text{descendant} \\
 & & \mid & \text{descendant-or-self} \\
 & & \mid & \text{ancestor} \\
 & & \mid & \text{ancestor-or-self} \\
 & & \mid & \text{following-sibling} \\
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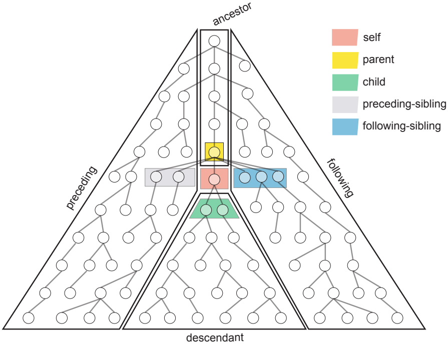
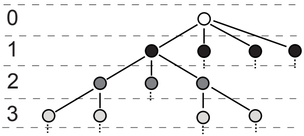
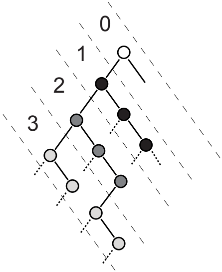


- Multi-directional tree navigation
- Node selection and path existence
- Almost full XPath

Models for XML Documents

- Finite ordered unranked trees, one label per node
- Bijective encoding of unranked trees as binary trees





- XML documents seen as finite ordered binary trees
  - Without loss of generality
  - XPath navigation must be expressed in binary style

Candidate Logics for XML

1. First-Order Logic (FO) and variants (over trees)
   - ✓ close to $\mathcal{L}_{\text{XPath}}$ expressive power
   - ✗ do not fully capture schemas
2. Monadic Second-Order Logic (WS2S)
   - ✓ extends FO with quantification over sets of nodes
   - ✓ captures $\mathcal{L}_{\text{XPath}}$ and finite tree automata
   - ✗ complexity for satisfiability: hyperexponential
   - ✗ blow-ups observable for XPath containment
3. Alternation-free fragment of the μ-calculus (AFMC)
   - ✓ supports schemas and XPath (when extended with *converse*)
   - ✓ complexity for satisfiability: $2^{O(n \cdot \log(n))}$ (with converse)
   - ✗ the solver a priori explores Kripke structures
   - ✗ formulas are more general than needed for XML
   - ✗ low performance in practice (does not scale to large instances)

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Contribution Idea

- design a specific logic whose models are finite trees
- design the algorithm for satisfiability-testing

Remark #1

- Finite tree models
  - avoid exploring useless models
  - allow a bottom-up algorithm for satisfiability-testing

Remark #2

- Only finite recursion is of interest for XPath and Schemas

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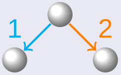
Formulas of the $\mathcal{L}_\mu$ Logic

- Programs $\alpha \in \{1, 2, \overline{1}, \overline{2}\}$ for navigating binary trees ($\overline{\overline{\alpha}} = \alpha$)



$$
\begin{array}{rclll}
\mathcal{L}_\mu \ni \varphi, \psi & ::= & & & \text{formula} \\
 & & \top & & \quad \text{true} \\
 & | & \sigma \quad | \quad \neg\sigma & & \quad \text{atomic prop (negated)} \\
 & | & \gamma^{\bullet} \quad | \quad \neg\gamma^{\bullet} & & \quad \text{context (negated)} \\
 & | & \varphi \vee \psi & & \quad \text{disjunction} \\
 & | & \varphi \wedge \psi & & \quad \text{conjunction} \\
 & | & \langle\alpha\rangle\,\varphi \quad | \quad \neg\,\langle\alpha\rangle\,\top & & \quad \text{existential (negated)} \\
 & | & X & & \quad \text{variable} \\
 & | & \mu X.\varphi & & \quad \text{unary fixpoint} \\
 & | & \mu\overline{X_i.\varphi_i} \text{ in } \psi & & \quad n\text{-ary fixpoint}
\end{array}
$$

- Closed formulas

Semantics of $\mathcal{L}_\mu$

- The set of models of a formula φ is the set of finite binary trees for which φ is satisfied on some node

- $\mu Z.\varphi$: finite recursion
- $\{\overline{1}, \overline{2}\}$ required for forward axes!
- $\{1, 2\}$ required for reverse axes!
- Converse programs are crucial
- $t_{\text{xpath}}(e, \chi) : \mathcal{L}_{\text{XPath}} \times \mathcal{L}_\mu \rightarrow \mathcal{L}_\mu$
- χ is the latest navigation step

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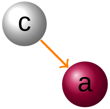

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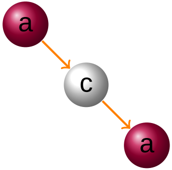

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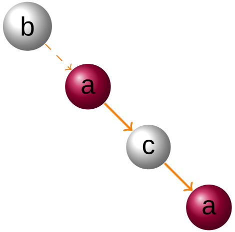

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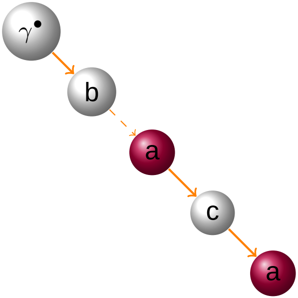

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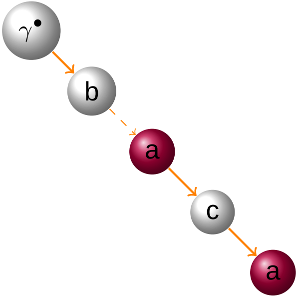

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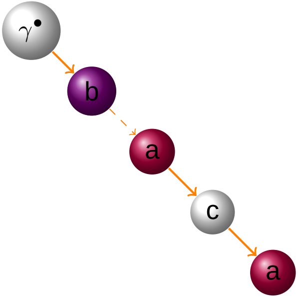

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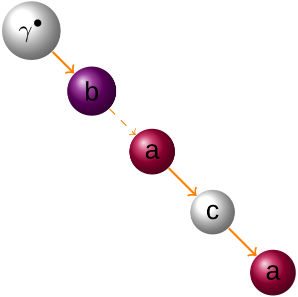

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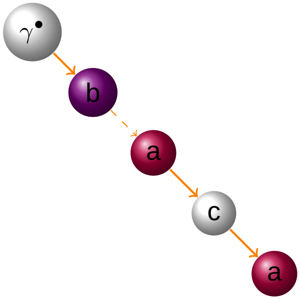

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XPath and Closure under Negation of $\mathcal{L}_\mu$

A Very Important Property

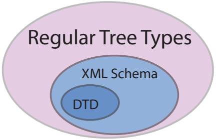
- $\mathcal{L}_{\text{XPath}}$ translations are never of the form: $\mu X . \langle \alpha \rangle X \vee \langle \overline{\alpha} \rangle X$
- $\mathcal{L}_{\text{XPath}}$ translations are always cycle-free
  - no occurrence of both a path and its converse between a fixpoint binder and its variable

- Restricting $\mathcal{L}_\mu$ to cycle-free formulas ensures closure under negation of recursion
  - The negation of finite recursion remains finite recursion
  - $\neg\varphi$ is expressible in $\mathcal{L}_\mu$ for all $\varphi \in \mathcal{L}_\mu$
    - Computable using De Morgan's laws, e.g. $\neg \langle \alpha \rangle \varphi = \neg \langle \alpha \rangle \top \vee \langle \alpha \rangle \neg\varphi$, and $\neg\mu X . \varphi = \mu X . \neg\varphi \left\{ {}^{\neg X}/_{X} \right\}$
- Crucial for implication (e.g., XPath containment)

Translating Schemas into $\mathcal{L}_\mu$

Models for Schemas

- Schema languages correspond to subclasses of *regular tree types* [Murata et al., 2005]



Translating Regular Tree Types into $\mathcal{L}_\mu$

- The binary encoding of trees also applies to tree types
- Binary tree type expressions model schemas without loss of generality
- They can be translated into the logic ($t_{\text{schema}}(\cdot) : \mathcal{L}_{\text{type}} \to \mathcal{L}_\mu$)
  - the *n*-ary fixpoint binder is used for mutually recursive definitions
  - only forward programs $\alpha \in \{1, 2\}$ are used

Formulating Decision Problems to be Solved

- $t_{\text{xpath}}(e, \chi) : \mathcal{L}_{\text{XPath}} \times \mathcal{L}_\mu \to \mathcal{L}_\mu$ and $t_{\text{schema}}(T) : \mathcal{L}_{\text{type}} \to \mathcal{L}_\mu$
- $\mathcal{L}_\mu$ closed under boolean operations

- XPath expressions $e_1, ..., e_n$ and schemas $T_1, ..., T_n$
- $\gamma^\bullet$ for comparing XPath expresssions from the same context

Many Decision Problems Can be Formulated

- XPath emptiness: $t_{\text{xpath}}(e_1, \gamma^\bullet \wedge t_{\text{schema}}(T_1))$
- XPath typing: $t_{\text{xpath}}(e_1, \gamma^\bullet \wedge t_{\text{schema}}(T_1)) \wedge \neg t_{\text{schema}}(T_2)$
  - if the formula is unsatisfiable then all nodes selected by e_1 under type constraint T_1 are included in the type T_2
- XPath containment:
  $t_{\text{xpath}}(e_1, \gamma^\bullet \wedge t_{\text{schema}}(T_1)) \wedge \neg t_{\text{xpath}}(e_2, \gamma^\bullet \wedge t_{\text{schema}}(T_2))$
- XPath equivalence, XPath overlap

Deciding $\mathcal{L}_\mu$ Satisfiability

- Does a formula $\psi \in \mathcal{L}_\mu$ admit a satisfying finite binary tree?

Principles

- Enumerate finite binary trees, look for a node on which ψ holds
- The truth status of a formula ψ can be determined from the status of a few of its subformulas
- The Fisher-Ladner Closure cl(ψ) $=$ $\{$ subformula of ψ where fixpoints are unwounded once $\}$
- We focus on a subset Lean(ψ) $\subseteq$ cl(ψ):
  - atomic propositions (alphabet symbols in ψ)
  - existential formulas

Example

Emptiness of XPath expression: `self::b/parent::a`

- $\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2 \rangle X$
- $\exp(\varphi) = b \vee \langle 2 \rangle \varphi$

$$\text{Lean}(\psi) = \left\{ \begin{array}{l} \langle 1 \rangle \top, \\ \langle \bar{1} \rangle \top, \\ \langle 2 \rangle \top, \\ \langle \bar{2} \rangle \top, \\ \sigma, \\ a, \\ b, \\ \langle 1 \rangle \varphi, \\ \langle 2 \rangle \varphi \end{array} \right\}$$

- The atomic proposition "σ" simulates an infinite alphabet ($\sigma \equiv \neg a \wedge \neg b$)

Nodes of the Searched Binary Tree

- The satisfiability-testing algorithm attempts to build a satisfying finite binary tree such that some node satisfies ψ

- A node is a ψ-type: a set $t \subseteq \text{Lean}(\psi)$ which satisfies constraints, e.g.:
  - modal consistency: $\forall \langle\alpha\rangle\,\varphi \in \text{Lean}(\psi), \langle\alpha\rangle\,\varphi \in t \Rightarrow \langle\alpha\rangle \top \in t$
  - tree node: $\left\langle\bar{1}\right\rangle \top \notin t \vee \left\langle\bar{2}\right\rangle \top \notin t$
  - labeled with exactly one atomic proposition $\sigma \in t$
- a ψ-type valuates any formula in $\text{cl}(\psi)$ via a relation $\dot{\in}$
  - for instance $\varphi_1 \wedge \varphi_2 \mathrel{\dot{\in}} t$ iff $\varphi_1 \mathrel{\dot{\in}} t$ and $\varphi_2 \mathrel{\dot{\in}} t$

Satisfiability-Testing Algorithm: Principles

Bottom-up Construction of a Tree of ψ-types

- A set T of ψ-types is repeatedly updated (least fixpoint computation)
  - Initially: $\emptyset$
  - Step 1 : all possible leaves are added
  - Step i : all possible parent nodes of current nodes are added

Termination

- If ψ is present in some root node, then ψ is satisfiable
- The algorithm returns a satisfying model as soon as it is found
- Otherwise, it terminates when no more node can be added
  - all roots of all buidable finite trees have been added

Example

$\text{Lean}(\psi) = \{\langle 1 \rangle \top,\ \langle \overline{1} \rangle \top,\ \langle 2 \rangle \top,\ \langle \overline{2} \rangle \top,\ \sigma,\ a,\ b,\ \langle 1 \rangle \varphi,\ \langle 2 \rangle \varphi\}$

$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2 \rangle X \equiv \exp(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

$\text{Lean}(\psi) = \{ \langle 1 \rangle \top,\ \langle \bar{1} \rangle \top,\ \langle 2 \rangle \top,\ \langle \bar{2} \rangle \top,\ \sigma,\ a,\ b,\ \langle 1 \rangle \varphi,\ \langle 2 \rangle \varphi \}$

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$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2 \rangle X \equiv \exp(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

$T^0 = \emptyset$

$$\text{Lean}(\psi) = \left\{\langle 1\rangle \top,\ \left\langle \overline{1}\right\rangle \top,\ \langle 2\rangle \top,\ \left\langle \overline{2}\right\rangle \top,\ \sigma,\ a,\ b,\ \langle 1\rangle \varphi,\ \langle 2\rangle \varphi\right\}$$

$\psi = a \wedge \langle 1\rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2\rangle X \equiv \exp(\varphi) = b \vee \langle 2\rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

$T^1 = ?$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{\langle 1 \rangle \top, \langle \bar{1} \rangle \top, \langle 2 \rangle \top, \langle \bar{2} \rangle \top, \sigma, a, b, \langle 1 \rangle \varphi, \langle 2 \rangle \varphi\}$

$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2 \rangle X \equiv \exp(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

$$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \quad \}$$

$$T^0 = \emptyset$$

$\text{Lean}(\psi) = \{ \langle 1 \rangle \top,\ \langle \bar{1} \rangle \top,\ \langle 2 \rangle \top,\ \langle \bar{2} \rangle \top,\ \sigma,\ a,\ b,\ \langle 1 \rangle \varphi,\ \langle 2 \rangle \varphi \}$

$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2 \rangle X \equiv \text{exp}(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

$T^2 = ?$

$T^1 = \{ \ \sigma \ \ \sigma \ \ \sigma \ \ a \ \ a \ \ a \ \ b \ \ b \ \ b \ \}$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{\langle 1\rangle \top,\ \langle \bar{1}\rangle \top,\ \langle 2\rangle \top,\ \langle \bar{2}\rangle \top,\ \sigma,\ a,\ b,\ \langle 1\rangle \varphi,\ \langle 2\rangle \varphi\}$

$\psi = a \wedge \langle 1\rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2\rangle X \equiv \exp(\varphi) = b \vee \langle 2\rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

Does $b\ \langle 2 \rangle\, \varphi$ belong to T^2 ?

$T^2 = ?$

$T^1 = \{\ \sigma\quad \sigma\quad \sigma\quad a\quad a\quad a\quad b\quad b\quad b\ \}$

$T^0 = \emptyset$

$\mathrm{Lean}(\psi) = \{\langle 1 \rangle \top,\ \langle \bar{1} \rangle \top,\ \langle 2 \rangle \top,\ \langle \bar{2} \rangle \top,\ \sigma,\ a,\ b,\ \langle 1 \rangle \varphi,\ \langle 2 \rangle \varphi\}$

$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2 \rangle X \equiv \exp(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

Does $b \, \langle 2 \rangle \, \varphi$ belong to T^2 ?

$T^2 = ?$

$T^1 = \{ \; \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \; \}$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{\langle 1 \rangle \top, \; \langle \bar{1} \rangle \top, \; \langle 2 \rangle \top, \; \langle \bar{2} \rangle \top, \; \sigma, \; a, \; b, \; \langle 1 \rangle \varphi, \; \langle 2 \rangle \varphi\}$

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Emptiness check of XPath expression `self::b/parent::a`

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Does $b \, \langle 2 \rangle \, \varphi$ belong to T^2 ?

$T^2 = ?$

$T^1 = \{ \; \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \; \}$

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$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2 \rangle X \equiv \exp(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

Does $b \, \langle 2 \rangle \varphi$ belong to T^2 ? Yes!

$T^2 = ?$

$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \}$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{\langle 1 \rangle \top, \langle \bar{1} \rangle \top, \langle 2 \rangle \top, \langle \bar{2} \rangle \top, \sigma, a, b, \langle 1 \rangle \varphi, \langle 2 \rangle \varphi\}$

$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2 \rangle X \equiv \exp(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

Does b satisfy ψ ?

$T^2 = ?$

$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \}$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{ \langle 1 \rangle \top, \ \langle \bar{1} \rangle \top, \ \langle 2 \rangle \top, \ \langle \bar{2} \rangle \top, \ \sigma, \ a, \ b, \ \langle 1 \rangle \varphi, \ \langle 2 \rangle \varphi \}$

$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X . b \vee \langle 2 \rangle X \equiv \exp(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

$T^2 = ?$ Does b satisfy ψ ? No!

$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \}$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{\langle 1 \rangle \top,\ \langle \bar{1} \rangle \top,\ \langle 2 \rangle \top,\ \langle \bar{2} \rangle \top,\ \sigma,\ a,\ b,\ \langle 1 \rangle \varphi,\ \langle 2 \rangle \varphi\}$

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Emptiness check of XPath expression `self::b/parent::a`

Example

Does $a \, \langle 1 \rangle \, \varphi$ belong to T^2 ?

$T^2 = ?$

$T^1 = \{ \sigma \;\; \sigma \;\; \sigma \;\; a \;\; a \;\; a \;\; b \;\; b \;\; b \}$

$T^0 = \emptyset$

$\mathrm{Lean}(\psi) = \{ \langle 1 \rangle \top, \; \langle \bar{1} \rangle \top, \; \langle 2 \rangle \top, \; \langle \bar{2} \rangle \top, \; \sigma, \; a, \; b, \; \langle 1 \rangle \varphi, \; \langle 2 \rangle \varphi \}$

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Emptiness check of XPath expression `self::b/parent::a`

Example

Does $a \, \langle 1 \rangle \, \varphi$ belong to T^2 ?

$T^2 = ?$

$T^1 = \{ \; \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \; \}$

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$\psi = a \wedge \langle 1 \rangle \varphi$ with $\varphi = \mu X . b \vee \langle 2 \rangle X \equiv \exp(\varphi) = b \vee \langle 2 \rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

Does a $\langle 1 \rangle \varphi$ belong to T^2 ?

$T^2 = ?$

$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \}$

$T^0 = \emptyset$

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Emptiness check of XPath expression `self::b/parent::a`

Example

Does $a \, \langle 1 \rangle \, \varphi$ belong to T^2 ? Yes!

$T^2 = ?$

$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \}$

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Emptiness check of XPath expression `self::b/parent::a`

Example

$T^2 = ?$

Does a satisfy ψ?

$T^1 = \{ \ \sigma \ \ \sigma \ \ \sigma \ \ a \ \ a \ \ a \ \ b \ \ b \ \ b \ \}$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{\langle 1\rangle \top,\ \langle \bar{1}\rangle \top,\ \langle 2\rangle \top,\ \langle \bar{2}\rangle \top,\ \sigma,\ a,\ b,\ \langle 1\rangle \varphi,\ \langle 2\rangle \varphi\}$

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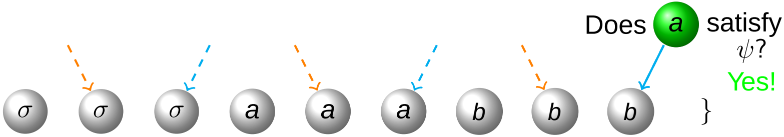
Emptiness check of XPath expression `self::b/parent::a`

Example

$T^2 = ?$

$T^1 = \{$



$\}$

$T^0 = \emptyset$

$\mathrm{Lean}(\psi) = \{\langle 1\rangle\top,\ \langle\bar{1}\rangle\top,\ \langle 2\rangle\top,\ \langle\bar{2}\rangle\top,\ \sigma,\ a,\ b,\ \langle 1\rangle\varphi,\ \langle 2\rangle\varphi\}$

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Emptiness check of XPath expression `self::b/parent::a`

Example

$$\left|T^2\right| = 81$$

$$T^1 = \{\ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b\ \}$$

$$T^0 = \emptyset$$

$\text{Lean}(\psi) = \{\langle 1\rangle \top,\ \langle \bar{1}\rangle \top,\ \langle 2\rangle \top,\ \langle \bar{2}\rangle \top,\ \sigma,\ a,\ b,\ \langle 1\rangle \varphi,\ \langle 2\rangle \varphi\}$

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Emptiness check of XPath expression `self::b/parent::a`

Example

$\longrightarrow$ return satisfiable!

$\left|T^2\right| = 81$

$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \}$

$T^0 = \emptyset$

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Emptiness check of XPath expression `self::b/parent::a`

Example

$T^3 = \ldots$

$\left|T^2\right| = 81$

$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \}$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{\langle 1\rangle \top,\ \langle \bar{1}\rangle \top,\ \langle 2\rangle \top,\ \langle \bar{2}\rangle \top,\ \sigma,\ a,\ b,\ \langle 1\rangle \varphi,\ \langle 2\rangle \varphi\}$

$\psi = a \wedge \langle 1\rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2\rangle X \equiv \exp(\varphi) = b \vee \langle 2\rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Example

$T^n = T^{n-1}$ (fixpoint)

$T^3 = \ldots$

$\left|T^2\right| = 81$

$T^1 = \{ \sigma \quad \sigma \quad \sigma \quad a \quad a \quad a \quad b \quad b \quad b \quad \}$

$T^0 = \emptyset$

$\text{Lean}(\psi) = \{\langle 1\rangle \top,\ \langle \bar{1}\rangle \top,\ \langle 2\rangle \top,\ \langle \bar{2}\rangle \top,\ \sigma,\ a,\ b,\ \langle 1\rangle \varphi,\ \langle 2\rangle \varphi\}$

$\psi = a \wedge \langle 1\rangle \varphi$ with $\varphi = \mu X.b \vee \langle 2\rangle X \equiv \exp(\varphi) = b \vee \langle 2\rangle \varphi$

Emptiness check of XPath expression `self::b/parent::a`

Correctness & Complexity

Theorem

The satisfiability problem for a formula $\psi \in \mathcal{L}_\mu$ is decidable in time $2^{O(n)}$ where $n = |Lean(\psi)|$.

Theorem

For $e \in \mathcal{L}_{XPath}$ and a regular tree type expression T, the translations of e and T in $\mathcal{L}_\mu$ are linear in the size of e and T.

Corollary

XPath decision problems (e.g., typing, containment, emptiness, equivalence) in presence of schemas can be decided in time complexity $2^{O(n)}$.

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For $e \in \mathcal{L}_{XPath}$ and a regular tree type expression T, the translations of e and T in $\mathcal{L}_\mu$ are linear in the size of e and T.

Corollary

XPath decision problems (e.g., typing, containment, emptiness, equivalence) in presence of schemas can be decided in time complexity $2^{O(n)}$.

Solver Implementation Techniques

Idea: Implicit Representation

- The T^is can be represented by boolean expressions
- Set-theoretic operations: composition of boolean expressions

- A set of ψ-types is encoded by a Binary Decision Diagram (BDD) [Bryant, 1986]

Critical Optimizations

- Conjunctive partitioning and early quantification (aims at composing smaller BDDs)
- Good initial order of BDD variables

Implementation

- System fully implemented (Java + Buddy C BDD library)
- Implementation available: `http://wam.inrialpes.fr/xml/`

Some Examples and Demo

- DTD of the W3C SMIL 1.0 recommendation

Question	Answer	Time (ms)
`/descendant::video` $\subseteq$ `/descendant::video[parent::seq]`?	no	125
`/descendant::audio[preceding-sibling::video]` $\neq \emptyset$?	yes	109
`child::switch[ancestor::head]` $\subseteq$ `descendant::switch`?	yes	105

- Demo

Main Contributions

A tree logic offering an interesting balance

- expressiveness: regular tree types + multi-directional navigation + finite recursion
- complexity for satisfiability: $2^{O(n)}$ where $n = |\text{Lean}(\psi)|$
- Compilation of main XML concepts: linear
- extensibility warranted (sublogic of the AFMC with converse)

A system for solving basic decision problems involving XPath queries and schemas

- The largest XPath fragment effectively treated so far for static analysis
- The system benefits from the boolean closure of the logic
- Efficient implementation in practice

Future Work

Extending the Tree Logic

- Decidable counting constraints
- Decidable data-value comparisons

Applications

- Static type-checking of XSLT, XQuery
- Optimization
- XML Security
- Checking integrity constaints in XML databases
- Query comparison in P2P networks

Thank you!

`http://wam.inrialpes.fr/xml/`

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Murata, M., Lee, D., Mani, M., and Kawaguchi, K. (2005).
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